



Language: **English**

Day: **1**

Friday, July 10, 2015

Problem 1. We say that a finite set $\mathcal{S}$ of points in the plane is *balanced* if, for any two different points A and B in $\mathcal{S}$, there is a point C in $\mathcal{S}$ such that $AC = BC$. We say that $\mathcal{S}$ is *centre-free* if for any three different points A , B and C in $\mathcal{S}$, there is no point P in $\mathcal{S}$ such that $PA = PB = PC$.

- (a) Show that for all integers $n \geq 3$, there exists a balanced set consisting of n points.
- (b) Determine all integers $n \geq 3$ for which there exists a balanced centre-free set consisting of n points.

Problem 2. Determine all triples (a, b, c) of positive integers such that each of the numbers

$$ab - c, \quad bc - a, \quad ca - b$$

is a power of 2.

(A power of 2 is an integer of the form 2^n , where n is a non-negative integer.)

Problem 3. Let ABC be an acute triangle with $AB > AC$. Let Γ be its circumcircle, H its orthocentre, and F the foot of the altitude from A . Let M be the midpoint of BC . Let Q be the point on Γ such that $\angle HQA = 90^\circ$, and let K be the point on Γ such that $\angle HKQ = 90^\circ$. Assume that the points A , B , C , K and Q are all different, and lie on Γ in this order.

Prove that the circumcircles of triangles KQH and FKM are tangent to each other.



Saturday, July 11, 2015

Problem 4. Triangle ABC has circumcircle Ω and circumcentre O . A circle Γ with centre A intersects the segment BC at points D and E , such that B, D, E and C are all different and lie on line BC in this order. Let F and G be the points of intersection of Γ and Ω , such that A, F, B, C and G lie on Ω in this order. Let K be the second point of intersection of the circumcircle of triangle BDF and the segment AB . Let L be the second point of intersection of the circumcircle of triangle CGE and the segment CA .

Suppose that the lines FK and GL are different and intersect at the point X . Prove that X lies on the line AO .

Problem 5. Let $\mathbb{R}$ be the set of real numbers. Determine all functions $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfying the equation

$$f(x + f(x + y)) + f(xy) = x + f(x + y) + yf(x)$$

for all real numbers x and y .

Problem 6. The sequence $a_1, a_2, \dots$ of integers satisfies the following conditions:

- (i) $1 \leq a_j \leq 2015$ for all $j \geq 1$;
- (ii) $k + a_k \neq \ell + a_\ell$ for all $1 \leq k < \ell$.

Prove that there exist two positive integers b and N such that

$$\left| \sum_{j=m+1}^n (a_j - b) \right| \leq 1007^2$$

for all integers m and n satisfying $n > m \geq N$.